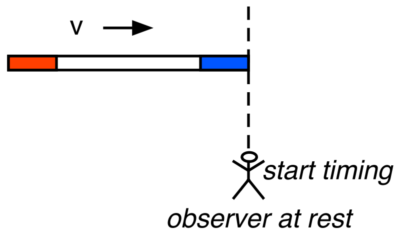
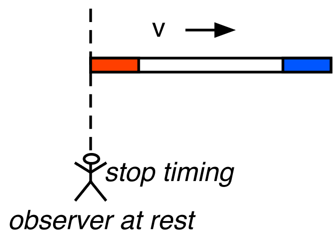
Length Contraction

Another surprising relativistic effect is the shortening of objects in the direction they are moving – which is called *length contraction*. It is easy to derive an expression for this contraction by thinking about an object traveling by and trying to measure its length. Since the object is moving, one cannot simply hold a meter stick up next to it. Instead, we will record the time it takes for the length of the object to pass by, and then knowing its speed, we can calculate the distance. The diagram below shows this situation.

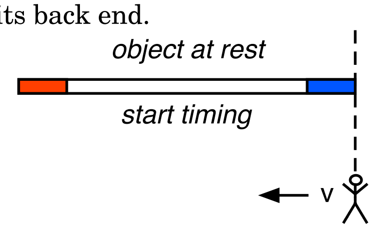


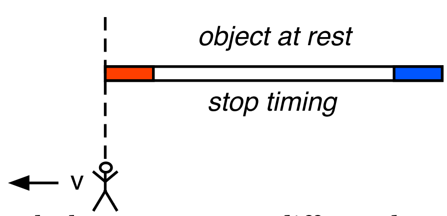


Notice that the time we just measured is a *proper time*, so we will call it t_0. It is a proper time because both events (the front of the object passing us and the back of the object passing us) occurred at the same location in our reference frame. Since the speed of the object is v, we can calculate the length L by saying

$$L = vt_0$$

Now let's switch to the frame of the object. It is at rest in its reference frame and sees us go by to the left, as shown below. The object would start timing when we pass by its front, and then stop timing when we pass by its back end.





The events are the same though they occur at two different locations according to the object. In this case, the object knows how long it is, which we will call L_0. The time it measured is a

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time dilated time, because the events were at different locations in its reference frame. According to the object we have

$$L_0 = vt$$

We know from time dilation the relationship between the two times, so we can do some algebra as follows

$$L = vt_0 = v\frac{t}{\gamma}$$

Substituting in our expression for L_0 we finally get

$$L = \frac{L_0}{\gamma}$$

The observer sees the object shorter than when at rest! It is important to note that this contraction is only in the direction of travel. Also, remember that these relativistic effects are always "happening to the other person," in that each reference frame thinks the other is shortened in the direction of travel and has time running slow.

The proper length (L_0) of an object is how long the object thinks it is. Anyone who is at rest relative to the object measures its proper length. You can think of the proper length of something as what you would measure if you stood next to it and used a tape measure. The one tricky example of a proper length is the distance between distant objects. As long as the two objects (two different stars perhaps) are not moving with respect to each other, their measured distance between them is a proper length. (Just think of it as a really long road through space.) When we say the closest star is a little over 4 light years away, that is a proper length.

If you are moving with respect to something, then you will measure a length contracted length, which will be shorter than its proper length. Because of the relative motion, you cannot stand next to it and use a tape measure.